Part 5 of 5:

A New MMP Method (Part 2)

(draft, 4 August 2008)

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This paper is the fifth part of a series of papers that can be downloaded here:

    http://m-schulze.webhop.net/schulze1.pdf
    http://m-schulze.webhop.net/schulze2.pdf
    http://m-schulze.webhop.net/schulze3.zip
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1. Introduction

In the fourth part of this series of papers, we presented a simple method to incorporate *proportional representation by the single transferable vote* (STV) into a *mixed member proportional representation* (MMP) scheme.

An MMP scheme is an election method where each voter has two votes: a *district vote* and a *party vote*. Let's say that the standard size of the parliament is 100 seats. Then the electorate is divided into districts; a certain number of seats (say 60 seats) is allocated to the districts; with the district vote, the voters choose the district winners. The party vote is used to determine how many seats each party deserves when <u>all 100 seats</u> are allocated by proportional representation on the national level. When a party wins X district seats and deserves Y > X seats according to its number of party votes, then this party gets, in addition to its district seats, Y - X extra seats. When a party wins X district seats and deserves Y < X seats according to its number of party votes, then this party keeps all these district seats, so that the total size of the parliament is increased by X - Y (*overhang seats*).

For the sake of simplicity, we ignored, in the fourth part of this series of papers, the most serious strategic problem of MMP methods: Under MMP methods, it is a useful strategy for a party A to run two lists (A1 and A2) and to tell its supporters to give their district votes to the candidates of list A1 and their party votes to list A2. When its supporters vote as they are told, then list A1 receives significantly more district seats than it deserves seats according to its number of party votes. These extra seats for list A1 are extra seats for party A, because list A2 already receives as many seats as party A deserves according to the proportional share of its number of supporters.

Frequently, *compensation seats* are used to eliminate the above mentioned strategic problem of MMP methods. That means: When a party wins overhang seats, then also the other parties get additional seats, so that the total size of the parliament increases further. These additional seats are allocated to the parties in such a manner that, in the end, each party has a (according to the numbers of party votes) proportional share of the seats.

However, the problem of this solution is that it is quite impossible to predict the size of the parliament. Example:

> In Albania, 140 of the 100 members of the Assembly are elected by *first-past-the-post* (FPP) in *single-member districts* (SMD). The other 40 seats are allocated to the parties on the national level as a part of an MMP scheme.

> In the 2005 general elections, the voters were told to give their district votes to the candidates of the two large parties (*Democratic Party* resp. *Socialist Party*) and to give their party votes to their smaller allies. With this strategy, the *Democratic Party* won 56 district seats with only 7.67% of the party votes; the *Socialist Party* won 42 district seats with only 8.89% of the party votes.

> If the maximum number of compensation seats was not limited, then the total size of the Assembly would have been around 730 seats (= 56 seats · 100% / 7.67%).

Therefore, we introduce in this paper a new concept to eliminate the above mentioned strategic problem of MMP methods:

For each party i, we define a *retain factor* $z_i \in \mathbb{R}$ with $0 \leq z_i \leq 1$. Suppose o_i is the number of voters who give their party vote to party i and do not cast a valid district vote. Suppose p_i is the number of voters who give their district vote to party i and do not cast a valid party vote. Suppose q_{ij} is the number of voters who give their district vote to party i and their party vote to party j. Then

$$\varepsilon_i := o_i + z_i \cdot p_i + \sum_j (z_i \cdot q_{ij} + (1 - z_j) \cdot q_{ji})$$

is the *effective number of party votes* for party i. In the beginning, $z_i := 0$ for all parties i. But when a party i wins overhang seats, then we increase z_i so that the effective number of party votes for party i increases while, for every other party j, the effective number of party votes for party j decreases. z_i is increased until, when the seats are allocated to the effective numbers of party votes, party i does not have any overhang seats anymore.

In other words: When a party i wins overhang seats, then a share $0 \leq z_i \leq 1$ of the party votes of those voters, who give their district vote to party i and their party vote to another party j, are counted as party votes for party i. This share is chosen in such a manner that the number of seats, party i deserves according to its new number of party votes, just equals the number of district winners of party i.

Where concrete numbers are needed, we use the elections to the *Berlin House of Representatives* (*Abgeordnetenhaus von Berlin*) to illustrate the proposed method. Currently, the electoral law says that the House consists of at least 130 members. 78 members (= 60%) are elected by *first-past-the-post* (FPP) in single winner districts (*single member plurality*, SMP); at least additional 52 members are elected by closed party lists to compensate party proportionality. However, as FPP does not lead to proportional results, usually significantly more than 52 additional members are needed to compensate party proportionality, so that the House usually has a size of about 150 members. We recommend that, in future, 115 of the 130 members (about 90%) should be elected by STV in districts of 8 to 22 seats. This is possible without creating too many additional members, because STV already leads to very proportional results.

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2. The District Vote

2.1. The Districts

Berlin is currently divided into 12 boroughs.

	borough	eligible voters (on 17 Sep. 2006)
1	Mitte	190,550
2	Friedrichshain-Kreuzberg	165,331
3	Pankow	274,380
4	Charlottenburg-Wilmersdorf	216,374
5	Spandau	160,411
6	Steglitz-Zehlendorf	213,787
7	Tempelhof-Schöneberg	231,249
8	Neukölln	193,014
9	Treptow-Köpenick	193,936
10	Marzahn-Hellersdorf	201,209
11	Lichtenberg	201,096
12	Reinickendorf	184,143
	total:	2,425,480

Table 2.1.1: The 12 Berlin boroughs

We recommend that the districts for the elections to the Berlin House of Representatives should be the 12 Berlin boroughs. When the Hill-Huntington method is being used to allocate the 115 district seats to the 12 districts, then we get two 8-seat districts (Friedrichshain-Kreuzberg, Spandau), five 9-seat districts (Mitte, Neukölln, Treptow-Köpenick, Lichtenberg, Reinickendorf), three 10-seat districts (Charlottenburg-Wilmersdorf, Steglitz-Zehlendorf, Marzahn-Hellersdorf), one 11-seat district (Tempelhof-Schöneberg), and one 13-seat district (Pankow).

2.2. The District Ballot

Each voter gets two ballots: a *district ballot* and a *party ballot*. Both ballots are on the same sheet of paper. See page 6.

The same candidate cannot run in more than one district. The same candidate cannot run simultaneously as an independent candidate and as a party candidate. The same candidate cannot run for more than one party simultaneously.

On the district ballot, the candidates are sorted according to their party affiliations. Candidates with the same party affiliation are sorted in an order determined by this party.

The individual voter ranks the candidates in order of preference. The individual voter may ...

... give the same preference to more than one candidate.

... keep candidates unranked. When a given voter does not rank all candidates, then this means (1) that this voter strictly prefers all ranked

candidates to all not ranked candidates and (2) that this voter is indifferent between all not ranked candidates.

... skip preferences. However, skipping some preferences does not have any impact on the result of the elections, since the result of the elections depends only on the order in which the individual voters ranks the candidates and not on the absolute preferences of the individual voters.

... give preferences to parties. When a given voter gives a preference to a party, then this means that each candidate of this party gets this preference unless this voter explicitly gives a different preference to this candidate.
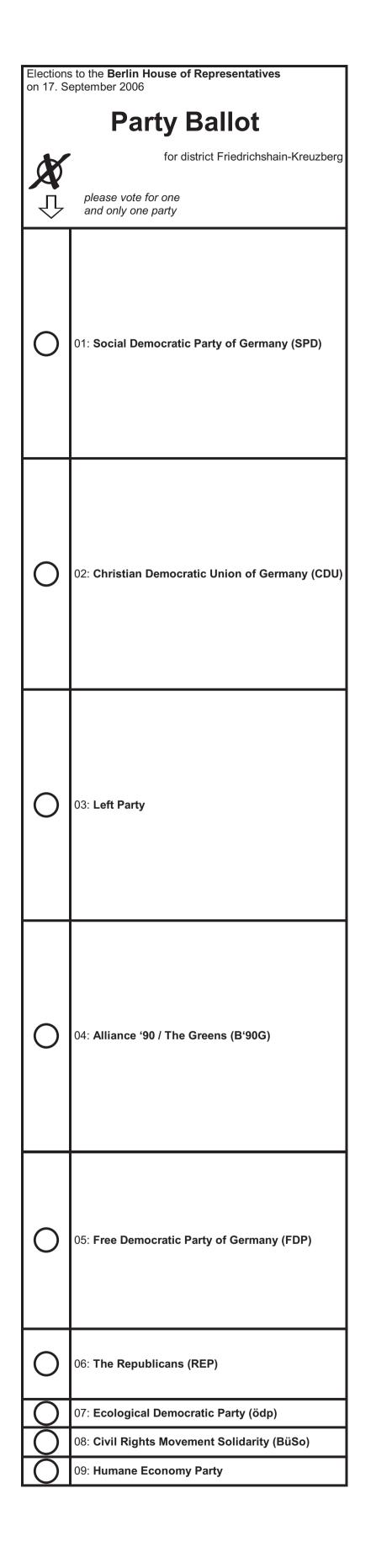
2.3. The District Winners

In each district, a proportional ranking of all candidates is calculated. At first, we calculate only the first M places of this proportional ranking, where M is the number of seats of the respective district. For the 8-seat district Friedrichshain-Kreuzberg, the first $M = 8$ places of this proportional ranking could look as follows:

1. Mutlu (B'90G, candidate 04.002) → elected (district vote)
2. Fischer (SPD, candidate 01.006) → elected (district vote)
3. Reinauer (Left, candidate 03.009) → elected (district vote)
4. Junge-Reyer (SPD, candidate 01.001) → elected (district vote)
5. Ratzmann (B'90G, candidate 04.001) → elected (district vote)
6. İzgin (Left, candidate 03.005) → elected (district vote)
7. Klotz (B'90G, candidate 04.003) → elected (district vote)
8. Samuray (CDU, candidate 02.004) → elected (district vote)

The idea is: If only the SPD supporters had participated, then this proportional ranking would have been Fischer, Junge-Reyer, etc.. If only the B'90G supporters had participated, then this proportional ranking would have been Mutlu, Ratzmann, Klotz, etc.. If only the Left Party supporters had participated, then this proportional ranking would have been Reinauer, İzgin, etc.. If only the CDU supporters had participated, then this proportional ranking would have been Samuray, etc..

As Friedrichshain-Kreuzberg is an 8-seat district, the first 8 candidates of this proportional ranking are elected.

District Ballot

for district Friedrichshain-Kreuzberg

*please rank the candidates
in order of preference* ⇩

01: **Social Democratic Party of Germany (SPD)**	
01.001: **Junge-Reyer**, Ingeborg	
01.002: **Zackenfels**, Stefan	
01.003: **Kitschun**, Susanne	
01.004: **Eggert**, Björn	
01.005: **Bayram**, Canan	
01.006: **Fischer**, Silke	
01.007: **Heinemann**, Sven	
01.008: **Miethke**, Petra	
01.009: **Kayhan**, Sevgi	
01.010: **Erdem**, Hediye	
01.011: **Klebba**, Sigrid	
01.012: **Postler**, Lorenz	
01.013: **Hehmke**, Andy	
01.014: **Dr. Beckers**, Peter	
01.015: **Lorenz**, Dorit	
01.016: **Borchard**, Andreas	
02: **Christian Democratic Union of Germany (CDU)**	
02.001: **Wansner**, Kurt	
02.002: **Bleiler**, Rainer	
02.003: **Ruhland**, Thomas	
02.004: **Samuray**, Sedat	
02.005: **Stry**, Ernst-Uwe	
02.006: **Rösner**, Helga	
02.007: **Glatzel**, Edgar	
02.008: **Schill**, Michael	
02.009: **Müller**, Götz	
02.010: **Freitag**, Jens-Matthias	
02.011: **Husein**, Timur	
02.012: **Wöhrn**, Marina	
02.013: **Taşkıran**, Ertan	
02.014: **Przewieslik**, Wolfgang	
02.015: **Konschak**, Benjamin	
02.016: **Bohl**, Daniel-Stephan	
03: **Left Party**	
03.001: **Michels**, Martina	
03.002: **Wolf**, Udo	
03.003: **Matuschek**, Jutta	
03.004: **Zillich**, Steffen	
03.005: **İzgin**, Figen	
03.006: **Günther**, Andreas	
03.007: **Vordenbäumen**, Vera	
03.008: **Krüger**, Wolfgang	
03.009: **Reinauer**, Cornelia	
03.010: **Bauer**, Kerstin	
03.011: **Mildner-Spindler**, Knut	
03.012: **Richter**, Claudia	
03.013: **Schüssler**, Lothar	
03.014: **Thimm**, Helga	
03.015: **Pempel**, Joachim	
03.016: **Sommer-Wetter**, Regine	
04: **Alliance '90 / The Greens (B'90G)**	
04.001: **Ratzmann**, Volker	
04.002: **Mutlu**, Özcan	
04.003: **Dr. Klotz**, Sibyll-Anka	
04.004: **Lux**, Benedikt	
04.005: **Herrmann**, Clara	
04.006: **Stephan**, André	
04.007: **Pohner**, Wolfgang	
04.008: **Dr. Altug**, Mehmet	
04.009: **Burkert-Eulitz**, Marianne	
04.010: **Kosche**, Heidi	
04.011: **Behrendt**, Dirk	
04.012: **Hauser-Jabs**, Christine	
04.013: **Schulz**, Franz	
04.014: **Kapek**, Antje	
04.015: **Wesener**, Daniel	
04.016: **Çetinkaya**, İstikbal	
05: **Free Democratic Party of Germany (FDP)**	
05.001: **Peters**, Frank	
05.002: **Dr. Hansen**, Nikoline	
05.003: **Eydner**, John	
05.004: **Hohl**, Heinrich	
05.005: **Salonek**, Gumbert-Olaf	
05.006: **Diener**, Thomas	
05.007: **Schaefer**, Martina	
05.008: **Wolf**, Tobias	
05.009: **Dr. Stolz**, Peter	
05.010: **Lauf**, Sebastian	
05.011: **Paun**, Christopher	
05.012: **Joecken**, Ilka	
06: **The Republicans (REP)**	
06.001: **Dr. Clemens**, Björn	
06.002: **Kuhn**, Daniel	
06.003: **Hinze**, Harald Björn Gunnar	
06.004: **Nestmann**, Günther	
07: **Ecological Democratic Party (ödp)**	
07.001: **Machel-Ebeling**, Johannes	
08: **Civil Rights Movement Solidarity (BüSo)**	
08.001: **Hinz**, Björn	
09: **Humane Economy Party**	
09.001: **Dr. Heinrichs**, Johannes	
10.001: **Eisner**, Udo (independent)	
11.001: **Stiewe**, Hauke (independent)	

Party Ballot

for district Friedrichshain-Kreuzberg

⇩ *please vote for one
and only one party*

◯ 01: **Social Democratic Party of Germany (SPD)**

◯ 02: **Christian Democratic Union of Germany (CDU)**

◯ 03: **Left Party**

◯ 04: **Alliance '90 / The Greens (B'90G)**

◯ 05: **Free Democratic Party of Germany (FDP)**

◯ 06: **The Republicans (REP)**

◯ 07: **Ecological Democratic Party (ödp)**

◯ 08: **Civil Rights Movement Solidarity (BüSo)**

◯ 09: **Humane Economy Party**

2.4. Allocation of the Voters to the District Winners

We want that a voter, who gave his district vote to a district winner of party a and who gave his party vote to another party b, does not get a double representation. Therefore, we will propose, in section 3.2, that, when party a wins more district seats that it deserves seats according to its number of party votes, then a share $z_a \in \mathbb{R}$ with $0 \leq z_a \leq 1$ of the party vote of each voter, who voted for a district winner of party a and who gave his party vote to another party b, should be counted as a party vote for party a and only $1 - z_a$ of this party vote should be counted as a party vote for party b. z_a is chosen in such a manner that the number of seats, party a deserves according to its new number of party votes, just equals the number of district winners of party a.

However, when the district winners are chosen by an STV method, then it is not immediately clear which voter voted for which district winner. Therefore, at first, we have to calculate, for each district separately, an allocation of the voters to the district winners.

Suppose $A_{\text{elected}} \subseteq A$ is the set of district winners. Suppose M is the number of district winners.

Suppose V is the list of voters i who satisfy at least one of the following two conditions:

    (1) Voter i casts a valid party vote.
    (2) Voter i is not indifferent between all the candidates of the set A_{elected}.

Those voters, who do not cast a valid party vote and are indifferent between all the candidates in A_{elected}, will be ignored. Suppose N is the number of voters in V.

Suppose $\lambda_{ij} \in \mathbb{R}$ is that share of voter $i \in V$ that is allocated to candidate $j \in A_{\text{elected}}$. Then λ_{ij} must satisfy at least the following three conditions:

$$(2.4.1) \qquad \forall\, i \in V \;\forall\, j \in A_{\text{elected}}: \qquad \lambda_{ij} \geq 0.$$

$$(2.4.2) \qquad \forall\, i \in V: \qquad \sum_{j=1}^{M} \lambda_{ij} = 1.$$

$$(2.4.3) \qquad \forall\, j \in A_{\text{elected}}: \qquad \sum_{i=1}^{N} \lambda_{ij} = N / M.$$

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Suppose $\forall\, i \in V\, \forall\, j \in A_{\text{elected}}$: $\eta_{ij} \in \mathbb{N}$ is the number of candidates $k \in A_{\text{elected}}$ with $k \succ_i j$. In other words: $\eta_{ij} \in \mathbb{N}$ is the number of candidates who are strictly preferred by voter i to candidate j.

Then the first approach would be to calculate λ_{ij} simply by minimizing $\sum_{i=1}^{N}\sum_{j=1}^{M}(\eta_{ij} \cdot \lambda_{ij})$ subject to (2.4.1) -- (2.4.3). In other words: We calculate λ_{ij} by minimizing the total number of votes that are transferred to a candidate j although the voter i strictly prefers another candidate k to candidate j.

The problem of this approach is that it usually leads to a large number of possible solutions for λ_{ij}, so that the final result of the election is not unique. Therefore, we use a more elaborated approach that guarantees that λ_{ij} is always unique. This approach says that λ_{ij} should satisfy (2.4.1) -- (2.4.3) and the following two conditions.

Condition #1:

Suppose the voters are sorted in such a manner σ that

$$\sum_{j \in A_{\text{elected}}}(\eta_{\sigma(1),j} \cdot \lambda_{\sigma(1),j}) \geq \sum_{j \in A_{\text{elected}}}(\eta_{\sigma(2),j} \cdot \lambda_{\sigma(2),j}) \geq \sum_{j \in A_{\text{elected}}}(\eta_{\sigma(3),j} \cdot \lambda_{\sigma(3),j}) \geq ...$$

Then this array should be as small as possible in the lexicographic sense.

That means:

Suppose also μ_{ij} satisfies (2.4.1) -- (2.4.3). Suppose the voters are sorted in such a manner τ that

$$\sum_{j \in A_{\text{elected}}}(\eta_{\tau(1),j} \cdot \mu_{\tau(1),j}) \geq \sum_{j \in A_{\text{elected}}}(\eta_{\tau(2),j} \cdot \mu_{\tau(2),j}) \geq \sum_{j \in A_{\text{elected}}}(\eta_{\tau(3),j} \cdot \mu_{\tau(3),j}) \geq ...$$

Then there should be **<u>no</u>** $1 \leq k \leq N$ with

1. $\sum_{j \in A_{\text{elected}}}(\eta_{\sigma(k),j} \cdot \lambda_{\sigma(k),j}) > \sum_{j \in A_{\text{elected}}}(\eta_{\tau(k),j} \cdot \mu_{\tau(k),j})$ and

2. $\forall\, 1 \leq i < k$: $\sum_{j \in A_{\text{elected}}}(\eta_{\sigma(i),j} \cdot \lambda_{\sigma(i),j}) = \sum_{j \in A_{\text{elected}}}(\eta_{\tau(i),j} \cdot \mu_{\tau(i),j})$.

Condition #2:

Suppose voter i is indifferent between candidate m and candidate n. Suppose voter j is indifferent between candidate m and candidate n. Then λ_{im} and λ_{in} should have the same ratio as λ_{jm} and λ_{jn}. In other words:

$\forall\, i,j \in V\, \forall\, m,n \in A_{\text{elected}}$: $(\,(\, m \approx_i n \text{ and } m \approx_j n\,) \Rightarrow (\, \lambda_{im} \cdot \lambda_{jn} = \lambda_{jm} \cdot \lambda_{in}\,)\,)$.

3. The Party Vote

3.1. The Party Ballot

On the party ballot of a given district, all those parties are listed that have nominated district candidates. The individual voter can vote for one and only one party. A party *qualifies* if and only if it has won at least one district seat or at least 5% of the valid party votes. Independent district winners are treated like parties with zero party votes.

3.2. Allocation of Seats to Parties

Suppose D_i is the number of district seats won by party i.

Suppose S_i is the number of seats that will have been allocated to party i during the allocation process.

Suppose p_i is the number of voters whose district votes are allocated to a district winner of party i and who do not cast a valid party vote.

Suppose q_{ij} is the number of voters whose district votes are allocated to a district winner of party i and who give their party vote to party j.

[$o_i = 0$ for all parties i because, as soon as a voter casts a valid party vote, his district vote is allocated to the district winners even if this voter is indifferent between all district winner.]

$z_i \in \mathbb{R}$ with $0 \leq z_i \leq 1$ is the *retain factor* of party i.

$$\varepsilon_i := z_i \cdot p_i + \sum_j (z_i \cdot q_{ij} + (1 - z_j) \cdot q_{ji})$$

is the *effective number of party votes* for party i. In the beginning, $z_i := 0$ for all parties i.

We recommend that the rules to allocate the seats to the parties should have the following properties:

- The seats are allocated according to the Sainte-Laguë method with 0.75 as first divisor. That means: The <u>effective</u> numbers of party votes for each party are divided by 0.75, 1.5, 2.5, 3.5, 4.5, 5.5, ... and the seats go to the largest quotients. [We presume for the rest of this paper that, when some quotients have the same value, then the seat goes to party i with larger D_i.]

- When a given party a wins more district seats than it deserves seats according to its <u>effective</u> number of party votes, then this party keeps all these district seats (*overhang seats*). To guarantee that the voters of the district candidates of party a do not get a double representation, we increase continuously the retain factor z_a of party a so that the <u>effective</u> number of party votes for party a increases, while the <u>effective</u> numbers of party votes for the other parties decrease. We increase z_a until (1) D_a is not larger anymore than the number of seats party

a deserves according to its <u>effective</u> number of party votes or (2) $z_a = 1$.

If $z_a = 1$, then this means, that it is not possible to achieve party proportionality simply by increasing z_a any further. Therefore, in this case, the other parties get additional seats to achieve party proportionality, so that the total size of the House increases (*compensation seats*). However, the total size of the House should not be larger than absolutely necessary to preserve party proportionality, because every additional seat compromises the idea that the House should be elected by proportional representation by the single transferable vote.

- However, to guarantee that the size of the House does not vary too much from one election to the other election, the minimum size is set to 131 members and the maximum size is set to 179 members. Therefore, we get $131 \leq \Sigma_j S_j \leq 179$.

- The constitution of Berlin says that a party has qualified if and only if it has received at least 5% of the valid party votes or has won at least one district seat. If a party has not qualified, then it must not get any seats (*threshold clause*).

- The total number of seats $\Sigma_j S_j$ should be odd (*stalemate clause*).

The Berlin House of Representatives should have at least 131 members. The seats should be allocated according to the Sainte-Laguë method with 0.75 as first divisor. Therefore, the numbers of <u>effective</u> party votes are divided by 0.75, 1.5, 2.5, 3.5, 4.5, 5.5, ... and the seats go to the 131 largest quotients. If each party i receives at least D_i seats, then the allocation procedure terminates; otherwise, we proceed as follows:

Step A (*calculation of the retain factors z_i*):

Stage 1:

> $r_i := \varepsilon_i / (D_i - 0.5)$ for each party i with $D_i > 1$.
> $r_i := \varepsilon_i / 0.75$ for each party i with $D_i = 1$.
> $r_i := \infty$ for each party i with $D_i = 0$.

> We take party a with minimum r_a.

> We increase z_a continuously. Simultaneously, we adjust ε_i for all parties i continuously. Simultaneously, we adjust continuously the 131 largest quotients that we get when we divide the effective numbers of party votes by 0.75, 1.5, 2.5, 3.5, 4.5, 5.5, etc..

> When we increase z_a continuously, then ε_a increases continuously, while ε_i decreases continuously for every other party i. Therefore, r_a increases continuously, while r_i decreases continuously for every other party i.

> [If there is more than one party a with minimum r_a, then we increase z_a for all these parties simultaneously in such a manner that r_a increases for all these parties equally.]

> We increase z_a continuously until at least one of the following three conditions is satisfied:

>> (3.2.1)   When the seats go to the 131 largest quotients, then each party i receives at least D_i seats.

>> (3.2.2) $z_i = 1$ for at least one party i.

>> (3.2.3)   The set of parties with minimum r_i has increased.

Stage 2:

> Repeat stage 1 until condition (3.2.1) or (3.2.2) is satisfied.

If condition (3.2.1) is satisfied, then the allocation procedure terminates and the seats go to the 131 largest quotients.

If condition (3.2.2) is satisfied and condition (3.2.1) is not satisfied, then this means, that the fact, that party a won more district seats than it deserves seats according to its number of party votes, cannot be caused exclusively by the above mentioned strategic problem of MMP methods. In this case, it is necessary to increase the number of compensation seats to preserve party proportionality. Therefore, we proceed as follows:

Step B (*allocation of seats to parties*):

Stage 1:

For each party i, we start with $s_i := D_i$.

Stage 2:

$r_i := \varepsilon_i / (D_i - 0.5)$ for each party i with $D_i > 1$.
$r_i := \varepsilon_i / 0.75$ for each party i with $D_i = 1$.
$r_i := \infty$ for each party i with $D_i = 0$.

$Y := \min_i r_i$.

[The idea is: As soon as the next quotient T, that will be rewarded with a seat, is equal to or strictly smaller than Y, party proportionality has been achieved, so that the allocation procedure can stop. Adding more seats would not improve proportionality according to party affiliations any further, but would worsen proportionality according to whichever other criteria the voters considered important when choosing the district winners by STV.]

Stage 3:

Repeat (until at least one of the termination conditions is satisfied):

$t_i := \varepsilon_i / (s_i + 0.5)$ for each party i with $s_i > 0$.
$t_i := \varepsilon_i / 0.75$ for each party i with $s_i = 0$.

$T := \max_i t_i$.
$U := \max_i \{ D_i \mid t_i = T \}$.

If at least one of the following two conditions is satisfied, then the allocation procedure terminates.

Condition #1:

The following three statements are satisfied:

(a) $\Sigma_j s_j \geq 131$.

(b) $\Sigma_j s_j$ is odd.

(c) $T \leq Y$.

Condition #2:

$\Sigma_j s_j = 179$.

If none of these two conditions is satisfied, then we proceed as follows:

The next seat goes to party i with $t_i = T$ and $D_i = U$. If there is more than one party with $t_i = T$ and $D_i = U$, then we decide randomly which party with $t_i = T$ and $D_i = U$ gets the next seat.

3.3. Allocation of the Party Seats to this Party's District Organizations

As candidates run on the district level and not on a city-wide level, we now have to allocate the seats of a party to this party's district organizations. We recommend that this allocation procedure should have the following properties:

- The seats are allocated according to the Sainte-Laguë method with 0.75 as first divisor.

- Each district organization must get at least as many seats as it has won district seats.

Therefore, we propose the following method to allocate the seats of party i to this party's district organizations:

Suppose D_i^j is the number of district seats won by party i in district j.
Suppose ε_i^j is the effective number of party votes for party i in district j.

Suppose S_i is the number of seats that have been allocated to party i as described in section 3.2.

s_i^j is the number of seats already allocated to the district organization of party i in district j.

Stage 1:

For each j, we start with $s_i^j := D_i^j$.

Stage 2:

Repeat (until $\Sigma_j s_i^j = S_i$):

$t_i^j := \varepsilon_i^j / (s_i^j + 0.5)$ for each district organization j of party i with $s_i^j > 0$.

$t_i^j := \varepsilon_i^j / 0.75$ for each district organization j of party i with $s_i^j = 0$.

$T := \max_j t_i^j$.
$U := \max_j \{ D_i^j \mid t_i^j = T \}$.

The next seat goes to district organization j with $t_i^j = T$ and $D_i^j = U$. If there is more than one district organization with $t_i^j = T$ and $D_i^j = U$, then we decide randomly which district organization with $t_i^j = T$ and $D_i^j = U$ gets the next seat.

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3.4. The Party Vote Winners

Now we calculate the remaining places of the proportional rankings of the districts. In each district j, we ignore all the candidates of parties i with $S_i^j = 0$.

Example: $S_{\text{SPD}}^{\text{FK}} = 3$, $S_{\text{Left}}^{\text{FK}} = 3$, $S_{\text{B'90G}}^{\text{FK}} = 5$, $S_{\text{CDU}}^{\text{FK}} = 1$. Then we ignore all the candidates of the FDP, the REP, the ödp, the BüSo, and the Humane Economy Party and the two independent candidates (Eisner, Stiewe), when we calculate the remaining places of the proportional ranking of the district Friedrichshain-Kreuzberg.

If $S_i^j > D_i^j$, then (in addition to those candidates who have already been elected by the district votes) those S_i^j - D_i^j candidates of this party are elected who are ranked highest in the proportional ranking of this district.

$D_{\text{SPD}}^{\text{FK}} = 2$ and $S_{\text{SPD}}^{\text{FK}} = 3$; therefore one ($= S_{\text{SPD}}^{\text{FK}}$ - $D_{\text{SPD}}^{\text{FK}}$) additional SPD candidate must be elected in the district Friedrichshain-Kreuzberg; this additional candidate is Eggert, because he is the highest ranked SPD candidate in the proportional ranking of this district who has not yet been elected. $D_{\text{Left}}^{\text{FK}} = 2$ and $S_{\text{Left}}^{\text{FK}} = 3$; therefore one ($= S_{\text{Left}}^{\text{FK}}$ - $D_{\text{Left}}^{\text{FK}}$) additional Left candidate must be elected in the district Friedrichshain-Kreuzberg; this additional candidate is Wolf, because he is the highest ranked Left candidate in the proportional ranking of this district who has not yet been elected. $D_{\text{B'90G}}^{\text{FK}} = 3$ and $S_{\text{B'90G}}^{\text{FK}} = 5$; therefore two ($= S_{\text{B'90G}}^{\text{FK}}$ - $D_{\text{B'90G}}^{\text{FK}}$) additional B'90G candidates must be elected in the district Friedrichshain-Kreuzberg; these additional candidates are Behrendt and Altug, because they are the highest ranked B'90G candidates in the proportional ranking of this district who have not yet been elected.

Therefore, we get:

1. Mutlu (B'90G, candidate 04.002) → elected (district vote)
2. Fischer (SPD, candidate 01.006) → elected (district vote)
3. Reinauer (Left, candidate 03.009) → elected (district vote)
4. Junge-Reyer (SPD, candidate 01.001) → elected (district vote)
5. Ratzmann (B'90G, candidate 04.001) → elected (district vote)
6. İzgin (Left, candidate 03.005) → elected (district vote)
7. Klotz (B'90G, candidate 04.003) → elected (district vote)
8. Samuray (CDU, candidate 02.004) → elected (district vote)
9. Eggert (SPD, candidate 01.004) → elected (party vote)
10. Heinemann (SPD, candidate 01.007)
11. Behrendt (B'90G, candidate 04.011) → elected (party vote)
12. Wolf (Left, candidate 03.002) → elected (party vote)
13. Altug (B'90G, candidate 04.008) → elected (party vote)
14. Bayram (SPD, candidate 01.005)
15. Kosche (B'90G, candidate 04.010)
16. Michels (Left, candidate 03.001)
17. Bleiler (CDU, candidate 02.002)
18. Herrmann (B'90G, candidate 04.005)
19. Zackenfels (SPD, candidate 01.002)
20. Vordenbäumen (Left, candidate 03.007)
21. etc.

4. Vacant Seats

When a seat gets vacant, then this seat goes to that candidate of this party who is ranked highest in the proportional ranking of this district. If the list of candidates of this party is exhausted or if this seat was the seat of an independent candidate, then this seat stays vacant.

Example: The seat of Klotz gets vacant. Then this seat goes to Kosche.

Therefore we get:

1. Mutlu (B'90G, candidate 04.002) → elected (district vote)
2. Fischer (SPD, candidate 01.006) → elected (district vote)
3. Reinauer (Left, candidate 03.009) → elected (district vote)
4. Junge-Reyer (SPD, candidate 01.001) → elected (district vote)
5. Ratzmann (B'90G, candidate 04.001) → elected (district vote)
6. İzgin (Left, candidate 03.005) → elected (district vote)
7. ~~Klotz (B'90G, candidate 04.003)~~ ~~→ elected (district vote)~~
8. Samuray (CDU, candidate 02.004) → elected (district vote)
9. Eggert (SPD, candidate 01.004) → elected (party vote)
10. Heinemann (SPD, candidate 01.007)
11. Behrendt (B'90G, candidate 04.011) → elected (party vote)
12. Wolf (Left, candidate 03.002) → elected (party vote)
13. Altug (B'90G, candidate 04.008) → elected (party vote)
14. Bayram (SPD, candidate 01.005)
15. Kosche (B'90G, candidate 04.010) → elected (successor of Klotz)
16. Michels (Left, candidate 03.001)
17. Bleiler (CDU, candidate 02.002)
18. Herrmann (B'90G, candidate 04.005)
19. Zackenfels (SPD, candidate 01.002)
20. Vordenbäumen (Left, candidate 03.007)
21. etc.